Journal of Engineering Physics and Thermophysics, Vol. 82, No. 3, 2009

OPTIMUM VALUES OF ENERGY CONVERTER EFFICIENCY

B. S. Fokin

UDC 536.7

Analytical expressions for optimum values of the thermodynamic and thermal efficiency of heat converters to other kinds of energy have been obtained with the use of the methods of nonlinear equilibrium thermodynamics. Their comparison with the data of industrial power plants has been made.

Keywords: *energy conversion, efficiency, nonequilibrium thermodynamics, optimum value, "golden section" number.*

Introduction. It is customary to estimate the efficiency of thermal power plants in which energy conversion occurs with the help of the so-called efficiencies. The theoretical efficiency η_t defines the efficiency of ideal thermal cycles with reversible thermodynamic processes. The real thermodynamic processes in energy converters proceed not in equilibrium and irreversibly with heat efficiencies η_h 35–40% lower than the ideal thermal coefficients η_t. The calculation of the heat efficiencies has been developed well for various thermal plants in analyzing reversible cycles in equilibrium thermodynamics. Determining the real heat efficiencies η_h calls for the application of methods of nonequilibrium thermodynamics of irreversible processes, from which the following analysis has been carried out in the linear approximation.

Interrelationship between the Heat, Thermal, and Thermodynamic Efficiencies of the Energy Converter. Any energy converter represents a thermodynamic system with inward and outward flows of different kinds of energy. For example, at the inlet to the power unit we have a heat flow, and at the outlet — electric current and voltage. From the dynamical point of view such a converter is a certain "blackbox" with nonequilibrium thermodynamic flow J_2 and thermodynamic force X_2 at the inlet and flow J_1 with force X_1 at the outlet (see Fig. 1).

According to the second principle and one of the postulates of nonequilibrium thermodynamics [1–7], the entropy production σ in the converter is a bilinear sum from the products of the flows and forces at the converter inlet and outlet:

$$\sigma = \frac{d_{\text{in}}S}{d\tau} = J_1\left(-\frac{U}{T_1}\right) + J_2\,\frac{T_1 - T_2}{T_1 T_2} \geq 0\,. \tag{1}$$

Note that the entropy production has a negative value at the outlet and a positive value at the inlet, but in the it is greater than or equal to zero (the latter holds for the equilibrium state). Therefore, to maintain the steady state of the thermodynamic system, the converter inlet should be energized due to the heat supply, and at the output it is necessary to allot the corresponding quantity of energy to the consumer.

By definition [5] the thermodynamic efficiency of the converter represents a positive ratio between the outlet and inlet production values:

$$\eta = -\frac{J_1 X_1}{J_2 X_2} = \frac{J_1 U T_2}{J_2\,(T_2 - T_1)}\,. \tag{2}$$

In the case of energy conversion by the Carnot ideal reversible cycle, the converter efficiency would only be restricted to the second principle of equilibrium thermodynamics and equal to the heat efficiency of the Carnot cycle [2]:

$$\eta_t = \frac{T_2 - T_1}{T_2}\,. \tag{3}$$

I. I. Polzunov Scientific-Production Association for investigation and design of power-generating plants, 3/6 Atamanskaya Str., St. Petersburg, 119167, Russia; email: fokbors@mail.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 82, No. 3, pp. 600–605, May–June, 2009. Original article submitted March 24, 2008.

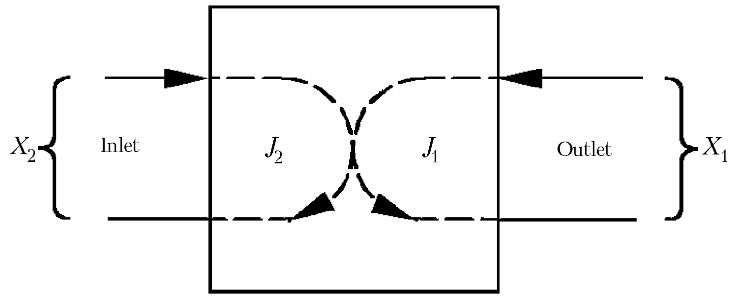

Fig. 1. Thermodynamical scheme of the energy converter in the form of a "blackbox."

From formulas (2) and (3) it follows that the heat efficiency of the considered energy converter is equal to

$$\eta_h = \frac{J_1 U}{J_2} = \eta_t \eta . \tag{4}$$

From formula (4) it follows that the energy conversion efficiency at given values of T_1 and T_2 is fully determined by the value of the thermodynamic efficiency η (2) depending on the irreversible losses resulting from the interaction of irreversible processes when heat is converted to electric energy.

Calculation of the Optimum Efficiency of the Energy Converter. The following analysis of the efficiency is carried out with the use of the linear phenomenological equations and the Onsager reciprocal relation [1–7]:

$$J_1 = L_{11} X_1 + L_{12} X_{12} ; \quad J_2 = L_{21} X_1 + L_{22} X_2 ; \quad L_{12} = L_{21} . \tag{5}$$

The phenomenological coefficients L_{ij} are constant and invariable for each particular energy converter and do not depend on J_i and X_i (i, j = 1, 2). The thermodynamics does not determine the numerical values of the coefficients L_{ij} but only restricts them to the following inequalities [1–3]:

$$L_{11} > 0 , \quad L_{22} > 0 , \quad L_{11} L_{22} \geq L_{12}^2 . \tag{6}$$

With the help of the system of equations (5) the thermodynamic efficiency (2) in [5] was transformed to the form

$$\eta = -jx = -\frac{q + zx}{q + (zx)^{-1}} . \tag{7}$$

Analysis shows that the domain of variability of q lies within the range from $q = 0$ (absence of interaction) to $q = 1$ (maximum possible interaction of inflow and outflow thermodynamic processes). In [5], it was established that at a given value of q = const the thermodynamic efficiency (7) has the maximum value

$$\eta_{max} = \left(\frac{q}{1 + \sqrt{1 - q^2}} \right)^2 . \tag{8}$$

The dimensionless relations between thermodynamic flows and forces thereby are equal in value and opposite in sign:

$$\left(\frac{j}{z} \right)_{max} \equiv \left(\frac{J_1}{J_2 z} \right)_{max} = -(xz)_{max} \equiv -\left(\frac{z X_1}{X_2} \right)_{max} = \sqrt{\eta_{max}} . \tag{9}$$

Formulas (8) and (9) define the first obligatory optimality condition of the thermodynamic efficiency η. Another condition is associated with the optimum value of the coefficient q in these formulas. The value of $q = q_{opt}$ lies in the range $0 \leq q_{opt} \leq 1$, since at $q = 0$ $\eta_{max} = 0$ (then $\eta_h = 0$), and at $q = 1$, while $\eta_{max} = 1$, the process of energy

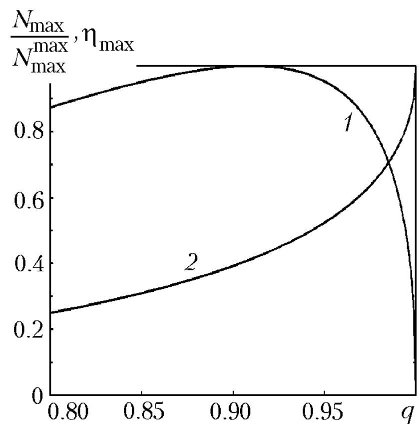

Fig. 2. Relative power at the outlet from the energy converter and its thermodynamic efficiency at maximum conjugation of thermodynamic flows and forces versus the conjugation parameter q: 1) calculation of (N_{max}/N_{max}^{max}) by Eqs. (14); 2) calculation of η_{max} by Eq. (8).

conversion becomes equilibrium-reversible ($\eta_h = \eta_t$), i.e., it is characterized by an unacceptably low power N of converted energy.

The thermodynamic forces are defined as follows:

$$X_1 = -\frac{U}{T_1}, \tag{10}$$

$$X_2 = \frac{T_2 - T_1}{T_1 T_2} = \frac{\eta_t}{T_1}. \tag{11}$$

Formulas (8)–(11), in view of the second equation of system (5) and the expression for the coefficient q in formula (7), permit defining the electric generator power at $\eta = \eta_{max}$ as

$$N_{max} = J_{1max} U_{max} = \frac{K(1-K)(\eta_t)^2 L_{22}}{(1+K)T_1}. \tag{12}$$

Analysis of formula (12) has shown that at a value of $K = K_{max} = 0.4142$ ($q_{max} = 0.91$) the electric power at the converter outlet acquires the maximum value

$$N_{max}^{max} = 0.1716 \frac{L_{22}}{T_1} (\eta_t)^2. \tag{13}$$

The electric power in the relative form is a function of the coefficient q alone (or $K = \sqrt{1-q^2}$):

$$\frac{N_{max}}{N_{max}^{max}} = 5.83 \frac{K(1-K)}{1+K} \equiv 5.83 \sqrt{1-q^2}\, \eta_{max}. \tag{14}$$

From the form of relations (8) and (14) it follows that in the range of $q_{max} = 0.91 \le q < 1.0$ the electric power at the converter outlet decreases with increasing q, and the thermodynamic efficiency determining the irreversible losses in the converter increases (see Fig. 2). The optimum value of $q = q_{opt}$ lies inside the above range and can be determined by finding the maximum dimensionless complex

$$E = \eta_{max} \left(\frac{N_{max}}{N_{max}^{max}} \right) = 5.83 \frac{K(1 - K)^2}{(1 + K)^2} \qquad (15)$$

by the parameter $K = \sqrt{1 - q^2}$. The E maximum is reached at

$$K_{opt} = (\sqrt{5} - 2) \approx 0.236 \quad \text{or} \quad q_{opt} = 2\sqrt{(\sqrt{5} - 2)} \approx 0.972\,. \qquad (16)$$

Formulas (8) and (16) determine the optimum values of the thermodynamic efficiency of any energy converter, i.e., the optimum irreversible losses at the optimum energy conversion rate:

$$\eta_{max}^{opt} = \frac{\sqrt{5} - 1}{2} \approx 0.618\,. \qquad (17)$$

It should be noted that the numerical value of the optimum thermodynamic efficiency fully coincides with the known in mathematics "golden section" number $(\sqrt{5} - 1)/2$ [8] which also determines the most harmonic proportions of architectural structures and paintings.

Thus, the optimum value of the heat efficiency of the energy converter is determined, according to formulas (4) and (17), by the simple dependence

$$\eta_h^{opt} \approx 0.618\eta_t\,. \qquad (18)$$

Comparison of the Calculated Efficiencies with the Data of Industrial Power Plants. The optimum heat efficiencies calculated by formula (18) were compared to those given in the literature and reference books [9–15] for nominal operating conditions of various power plants. The thermal efficiencies η_t entering into formula (18) were calculated for these power plants by the following relations:

steam-turbine plants of thermal and nuclear power stations (STPs of TPSs and NPSs):

$$\eta_t = 1 - \frac{T_1}{T_2}\,;$$

gas-turbine plants (GTPs):

$$\eta_t = 1 - \beta^{-\frac{k-1}{k}}\,;$$

internal combustion engines (ICEs) with a Trinkler cycle:

$$\eta_t = 1 - \frac{(\lambda\rho^k - 1)\,\varepsilon^{-(k-1)}}{\lambda - 1 + k\lambda(\rho - 1)}\,.$$

The results of the comparison between the calculated and real values of the efficiency are given graphically in Fig. 3.

The overwhelming majority of the real values of η_h lie near the calculated dependence (18) with a maximum deviation of ±15% from the optimum regime with η_h^{opt}. If this deviation is not associated with the error in determining η_h, then for different power plants it can be explained by the following reasons. Power units with $\eta_h > \eta_h^{opt}$ operate, apparently, under power underloaded conditions compared to the optimal regime. In this case, the parameter $q > q_{opt} = 0.972$ and the thermodynamic efficiency $\eta > \eta_{opt}$ (see Fig. 2). The value of η_h thereby becomes larger than η_h^{opt}, and the electrical load at the converter outlet is less than the optimum value. Calculations show that as η_h increases by 8% compared to η_h^{opt}, the electric power, i.e., the energy conversion rate, decreases by 9%. Such power units are more economical ($\eta_h > \eta_h^{opt}$) but have a lower capacity ($N_{max} < N_{max}^{opt}$).

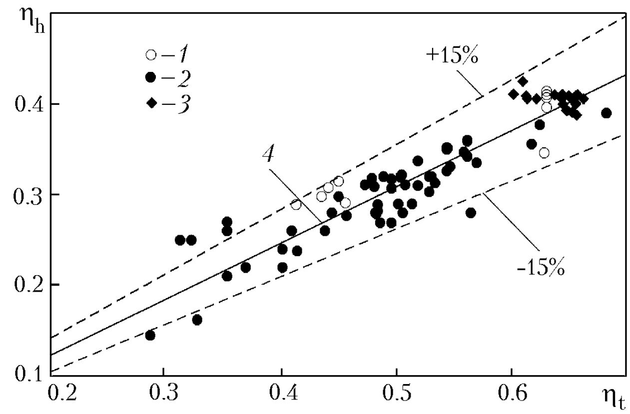

Fig. 3. Comparison of the real heat efficiencies of power plants (1, 2, 3) to the calculated optimal thermal efficiencies (4): 1) STPs of TPSs and NPSs [9, 12]; 2) GTP [10–14]; 3) ICE [15]; 4) calculation of η_h^{opt} by (18).

Analogous reasoning leads to the conclusion that an increase in the power of the energy converter to a value higher than the optimum value $N_{max} > N_{max}^{opt}$ will cause a decrease in its heat efficiency, since the irreversible losses will increase thereby and the thermodynamic efficiency will become lower than the optimum $\eta_{max}^{opt} \cong 0.618$.

Indeed, in analyzing formula (14) it has been shown that the maximum power at the outlet from the energy converter is attained for the parameter $q_{max} = 0.91 \le q_{max}^{opt} = 0.972$ at which the thermodynamic and, consequently, the heat efficiency are lower than the corresponding efficiencies in the optimal regime. Therefore, power units with a lower efficiency, compared to the optimum efficiency, have a higher capacity but are less economic. However, lower efficiencies $\eta_h < \eta_h^{opt}$ can be due to not only the excess forcing of the power of a unit but also to the choice of its parameters, the imperfection of the design, and thermohydraulic schemes, which leads to an increase in irreversible losses. In this case, it is necessary to fulfill the optimality condition (9) for the relation of thermodynamic flows and forces at the energy converter inlet and outlet and improve the design and the thermohydraulic characteristics of the energy converter, striving for optimum interaction of nonequilibrium processes at a conjugation coefficient $q_{max}^{opt} = 0.972$.

Conclusions. Optimizing the power of the energy converter with account for the irreversible losses arising in it has made it possible to determine the optimum value of the thermodynamic efficiency equal to the "golden section" number $(\sqrt{5} - 1)/2 \cong 0.618$. The optimum value of the heat efficiency η_h^{opt} is determined thereby by relation (18). It has been shown that a large number of industrial power plants operate with $\eta_h = \eta_h^{opt}$, i.e., in the optimal regime. Power plants with $\eta_h > \eta_h^{opt}$ operate under power underloaded conditions, and those with $\eta_h < \eta_h^{opt}$ operate either with power overloading or with a nonoptimal choice of parameters, imperfection of the design, and a bad organization of the thermohydraulic processes.

NOTATION

J, thermodynamic flow at the energy converter inlet (W) or outlet (A); $j = J_1/J_2$, relation between thermodynamic processes, V^{-1}; $k = 1.4$, adiabatic index for ideal diatomic gas; L, phenomenological components in the Onsager equation; N_{max}, electric power at a maximum value of η_{max}, W; N^{max}, electric power at a maximum value of q_{max}, W; $q = L_{12}/\sqrt{L_{11}L_{12}}$, conjugation coefficient determining the degree of interaction of nonequilibrium processes inside the "blackbox" — the energy converter; S, entropy, J/K; T, temperature, K; U, electric voltage, V; X_2, X_1, thermodynamic force at the energy converter inlet (K^{-1}) and outlet ($V \cdot K^{-1}$); $x = X_1/X_2$, relation between thermodynamic forces, V; $z = \sqrt{L_{11}L_{22}}$, stoichiometric coefficient, V^{-1}; β, degree of increase of pressure in the GTP compressor; Δ, finite change in the parameter; ε, degree of adiabatic compression of the gas in the cylinder; η, thermodynamic efficiency; η_t, thermal efficiency; η_h, heat efficiency; λ, degree of increase of gas pressure at isochoric heat input; ρ, degree of preliminary expansion of the gas at isobaric heat input; σ, entropy production inside the energy converter, W/K; τ, time, sec; E, dimensionless complex characterizing the power and irreversible losses at energy conversion. Subscripts: 1, 2, at the energy converter outlet and inlet, respectively; in, inside the energy converter; max, maximum; opt, optimum; h, heat; t, thermal.

REFERENCES

1. S. De. Groot and P. Mazur, *Non-Equilibrium Thermodynamics* [Russian translation], Mir, Moscow (1964).
2. I. Prigogine and D. Kondepudi, *Modern Thermodynamics. From Heat Engines to Dissipative Structures* [Russian translation], Mir, Moscow (2002).
3. I. Prigogine, *Introduction to Thermodynamics of Irreversible Processes* [Russian translation], Rand C Dynamics, Moscow–Izhevsk (2002).
4. I. Gyarmati, *Nonequilibrium Thermodynamics* [Russian translation], Mir, Moscow (1974).
5. S. R. Caplan and A. Essig, *Bioenergetics and Linear Nonequilibrium Thermodynamics* [Russian translation], Mir, Moscow (1986).
6. E. P. Ageev, *Nonequilibrium Thermodynamics in Questions and Answers* [in Russian], Éditorial URSS, Moscow (2001).
7. V. A. Zhuravlev, *Thermodynamics of Irreversible Processes in Problems and Solutions* [in Russian], Izd. Dom "Udmurtskii Universitet," Moscow–Izhevsk (1998).
8. Yu. A. Stepanov, F. S. Dem'yanyuk, A. A. Znamenskii, et al., *Brief Polytechnic Dictionary* [in Russian], Gostekhteoretizdat, Moscow (1955).
9. A. N. Kovalenko, *System Analysis and Optimization of the Characteristics of Thermal Processes in High-Power Plants*, Author's Abstract of Doctoral Dissertation (in Engineering), Central Boiler and Turbine Institute, St. Petersburg (1999).
10. L. V. Arsen'ev, V. G. Tyryshkin, I. A. Bogov, et al. (L. V. Arsen'ev and V. G. Tyryshkin Eds.), *Stationary Gas-Turbine Plants* [in Russian], Mashinostroenie, Leningrad (1989).
11. M. M. Kovalevskii, *Open-Cycle Stationary Gas-Turbine Plants* [in Russian], Mashinostroenie, Moscow (1975).
12. V. N. Yurenev and P. D. Lebedev (Eds.), *Heat-Engineering Handbook* [in Russian], Vol. 1, Énergiya, Moscow (1985).
13. G. G. Ol'khovskii, *Gas-Turbine Power Plants* [in Russian], Énergoatomizdat, Moscow (1985).
14. A. S. Lebedev, *Development of a Domestic Medium-Power Gas-Turbine Plant with the Application of a Complex of Modern Computational- Experimental Methods*, Author's Abstract of Doctoral Dissertation (in Engineering), St. Petersburg State University, St. Petersburg (2007).
15. V. A. Vainsheidt, I. I. Ivanchenko, and L. K. Kollerov (Eds.), *Diesels: Handbook* [in Russian], Mashinostroenie, Leningrad (1977).